



# Math League News

■ **Our Calculator Rule** Our contests allow both the TI-89 and HP-48. You may use any calculator without a QWERTY keyboard.

■ **Use the Internet to View Scores or Send Comments** to [comments@mathleague.com](mailto:comments@mathleague.com). You can see your results at [www.mathleague.com](http://www.mathleague.com).

■ **Upcoming Contest Dates & Rescheduling Contests** Contest dates (and alternate dates), all Tuesdays, are February 22 (February 15) and March 22 (March 15). If **vacations, school closings, or special testing days** interfere, please reschedule the contest. Attach a brief explanation, or scores will be considered unofficial. We sponsor an *Algebra Course I* Contest and contests for grades 4, 5, 6, 7, and 8. Get information and sample contests at [www.mathleague.com](http://www.mathleague.com).

■ **2011-2012 Contest Dates** We schedule the six contests to be held four weeks apart (mostly) and to end in March. Next year's contest (and alternate) dates, all Tuesdays, are October 18 (Oct 11), November 15 (Nov. 8), December 13 (Dec. 6), January 10 (Jan 3), February 14 (Feb. 7), March 13 (March 6). Have a testing or other conflict? Now is a good time to put an alternate date on calendar!

■ **What Do We Publish?** Did we not mention your name? We use everything we have when we write the newsletter. But we write the newsletter early, so sometimes we're unable to include items not received early enough. We try to be efficient! Sorry to those whose solutions were too "late" to use.

■ **T-Shirts Anyone?** We're often asked, "are T-shirts available? The logo lets us recognize fellow competitors!" Good news — we have MATH T-shirts in a variety of sizes at a **very** low price. Use them as prizes for high or even perfect scores, or just to foster a sense of team spirit! The shirts are of grey material and feature a small, dark blue logo in the "alligator region." A photo of the shirt is available at our website. There's one low shipping charge per order, regardless of order size. To order, use our website, [www.mathleague.com](http://www.mathleague.com).

■ **Contest Books Make A Great Resource** Have you seen our contest books? Kids love to work on past contests. To order, use our website, [www.mathleague.com](http://www.mathleague.com).

■ **General Comments About Contest #4:** Lynn Clark said, "This was the hardest one yet." James Conlee said, "Tough contest. Definitely should thin the herd." Robert W. Nielsen said, "This was so far the most challenging test of the year for us. Great questions." Robert Morewood said, "Thanks for another stimulating contest." Deborah Stepelman said, "Most questions seemed pretty difficult for a contest #4." Dan Halter said, "Our school has 'winterim' and most of our 'regular' students were unavailable to take this test." Cindy Crenshaw said, "Thanks for providing a first-rate set of contest problems that are accessible to a wide range of students. Our kids enjoy the challenges you pose and learn a lot in the process." Jon Graetz said, "Overall, this was a good contest, I thought." Susan Wong said, "Tough questions!" Sal Muciño said, "The last two competitions have been very challenging ... Thanks for the experience."

■ **Question 4-3: Comment and Appeal (Denied)** James Conlee said it was "difficult to understand what [#3] was asking." Ross Arseneau appealed on behalf of one of his students on the theory that the phrase "at least how many dogs" could be interpreted as allowing for additional answers greater than 7, for example 11. The Appeals Committee has reviewed this appeal and ruled that the phrase "at least" as used in this question means minimum number, and further that the alternative interpretation suggested would preclude a unique answer to the question and thus is not acceptable. Therefore 11, or any other number greater than 7, is an incorrect response.

■ **Question 4-4: Alternate Solution and Appeals (Denied)** Robert Morewood suggested an alternative solution to Question 4-4, stating that it "can be solved with basic trigonometry (for those who don't like solving equations that look quadratic). Find the two acute angles (using the inverse tangent) for the large right triangle formed by half the paper. The difference is an angle of the right triangle whose hypotenuse is the desired length. Find that length using the cosine of this difference angle. (Students who know enough trig identities can even complete this without a calculator.)" Robert W. Nielson, though he believed it to be incorrect,

appealed on behalf of a student who answered 8.19999. David Holze similarly appealed on behalf of a student who answered 8.199999999. These answers are in fact incorrect, as the correct answers with the same numbers of significant digits would be 8.20000 and 8.200000000 respectively.

■ **Question 4-5: Comments** Robert Morewood said, "I was pleased with many of our junior students, whose algebra is not yet well-developed, but nonetheless recognized that 2011 is a prime number and hence were able to answer #5. Later, they were able to follow the algebra in the official solution to discover that this answer is indeed unique." Rhonda de la Mar commented along the same lines that, "several of my students who were puzzled by an algebraic approach to #5 were thrilled that [since] we had entered a new year, they could try 2011 as an answer (and it worked)."

■ **Question 4-6: Comments, Appeals (Denied), and Alternative Solutions** John Cocharo said, "Boy that was the hardest problem that I have ever seen on the contest! Oh well!" James Conlee said, "#6 was just evil." We heard from many advisors who found the wording of Question 4-6 difficult or confusing; many of these advisors were appealing on behalf of students who had answered with polynomials of degrees greater than 3. Among those bringing this issue to our attention were Caleb McArthur, Paul Goldstein, Sal Muciño, Judson Ford, Silva Chang, Erin Best, Sean Kaiser, Barbara Gerson, Barbara Elliot, Don Barry, Dennis Gournic, Cindy Crenshaw, Jonathan Chen, Andrea Westgate, Suzanne Antink, and John Bartlett. The wording of the question calls for the polynomial of "least degree," allowing for only the single 3rd degree polynomial that is the official answer, as opposed to "AT least degree," which would have allowed for multiple answers of higher degrees. Perhaps the wording could have been better, but it does mandate that any answer of degree greater than 3 be considered incorrect. Several of our advisors, including Jon Graetz, Edward Groth, and Robert Morewood, suggested a simple trial-and-error approach to the question focused on an examination of the powers of  $\sqrt{3} + \sqrt{2}$  to see what might work. Squaring this key expression yields a term involving  $\sqrt{6}$ , which is problematic. Cubing the expression instead yields terms involving only  $\sqrt{3}$  and  $\sqrt{2}$ , so an acceptable linear combination of the resulting cube and the original expression is easy to find. Jon Graetz also suggested a solution in which  $\sqrt{3} + \sqrt{2}$  is plugged into a series of polynomials (with undetermined coefficients) set equal to  $\sqrt{3} - \sqrt{2}$ , progressing by degree until an acceptable solution is found. Neither the first degree polynomial ( $P(x) = a + bx$ ) nor the second degree polynomial ( $P(x) = a^2 + bx + c$ ) works, but the third degree polynomial  $P(x) = ax^3 + bx^2 + cx + d$  yields the correct answer. Professor M. Selby of the University of Windsor pointed out that since  $P(x) = x^4 - 10x^2 + 1 = 0$  and  $f(x) = x^3 + 10x = \sqrt{3} - \sqrt{2}$  when  $x = \sqrt{3} + \sqrt{2}$ , all of the polynomials that would satisfy this question are of the form  $Q(x)P(x) + f(x)$ , with  $Q(x)$  any polynomial with integral coefficients. All of the alternate answers (of unacceptably high degree) that were submitted can be generated from this relationship.

## Statistics / Contest #4

Prob #, % Correct (all reported scores)

4-1	72%	4-4	35%
4-2	63%	4-5	33%
4-3	50%	4-6	6%